Theoretical Analysis of Vacuum Evacuation in Viscous Flow and Its Applications

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The vacuum is classified into three categories depending on the state of gas: viscous, intermediate, and molecular flows. The viscous flow evacuation is generally regarded as a basic technique, with which pressure decreases along with the exponential curve of time. However, this is not always the case when the conductance of the pipe is considered.
To begin with, the author theoretically solved the evacuation equation in the viscous flow. In addition, he introduced the notion of "transferring pressure Π," calculated as S_0/C_C; S_0 refers to the pumping speed, while C_C refers to the ratio of the conductance of the pipe to the pressure. By examining the two extreme cases, the author has revealed a new fact: (a) when pressure >>Π, the evacuation curve is exponential as generally stated; whereas (b) when pressure << Π, the curve is not exponential but inversely proportional to time. In this paper, the author describes the details of his study and the applications of case (b).

Keywords: vacuum, viscous flow, Knudsen number, conductance, evacuation time

1. Introduction

The vacuum is defined as "the state of the space filled with the gas, the pressure of which is lower than that of the atmosphere" in JIS Z 8126 (Vacuum technology -Vocabulary). The vacuum technologies have found a wide variety of industrial applications depending on the pressure level (**Table 1**).

Table 1. Vacuum technologies classified by the pressure

Technology	Pressure(Pa)	Utilized physical phenomena
Vacuum suction	$10^5 \sim 10^4$	Pressure difference with the atmosphere
Vacuum distillation	$10^4 \sim 10^3$	Boiling point drop
Vacuum gas exchange	$10^1 \sim 10^{-3}$	Lower residual impurities
Vacuum evaporation	$10^{-2} \sim 10^{-7}$	Longer mean free path of the gas
Molecular beam epitaxy	$10^{-7} \sim 10^{-9}$	Lower impingement rate of residual molecules, for better grown films

Table 1 shows that the pressure of vacuum ranges over 10 to the 14th power, and that the technologies, equipment structure and specifications, tasks to be solved, and know-how needed for the specific range differs from one another.

In this paper, the author first conducted a theoretical analysis of vacuum evacuation in the viscous flow, which is usually regarded as a basic technique, and so not much is mentioned in general vacuum texts. Next he deduced the exact solution of the equations, and found answers in two extreme cases.

The two cases are: 1) in the case when the pipe's resistance can be neglected, 2) in the case when the pipe's resistance dominates the phenomena.

The author also introduced the notion of "transferring pressure" and showed that the general evacuation curve, which includes case 1) and 2), was explained totally. The result of the analysis of case 2), which is not mentioned usually, is very simple and also interesting. He also mentioned its applications and attentions at the last.

2. Categories of Vacuum

2-1 Categories by the pressure

JIS Z 8126 defines the pressure ranges of vacuum as **Table 2**.

Table 2. Pressure ranges of vacuum(1)

Vacuum categories	Pressure ranges (Pa)
Low vacuum	Atmosphere ~ 100
Medium vacuum	100 ~ 0.1
High vacuum	$0.1 \sim 10^{-5}$
Ultra high vacuum	Less than 10^{-5}

The author must note that these categories are easy to observe and also easy to understand intuitively, but do not represent actual physical phenomenon (kinetic state of gas molecules) and so are inadequate to physical analyses.

2-2 Categories by the kinetic state of gas

In order to analyze vacuum phenomena, categories by the kinetic (flow's) state of gas are necessary. Its boundaries are actually inexact, but it is usually classified into 3 categories in **Table 3**.

Table 3. Vacuum categories by the kinetic state of gas

Categories	Sub class	State of gas molecules	Criteria(3)
Viscous flow	Turbulent flow	Collision rate between molecules are much higher than that of molecules and wall	K<0.01, Re>2200
	Laminar flow		K<0.01, Re<1200
Interme-diate flow		Transient region of viscous and molecular flow	0.01<K<0.3
Molecular flow		Collision rate between molecules are much lower than that of molecules and wall	K>0.3

K : Knudsen number: Index of viscous/molecular flow.
K = λ/D, λ: mean free path (m), D: diameter (m)
Re : Reynolds number: Index of turbulent/laminar flow.
Re = Dvρ/η, D: diameter (m), v: flow speed (m/s),
ρ: density (kg/m³), η: viscosity (Pa·s)

The reason why these categories are important is that 'conductance,' which is an important index of vacuum evacuation, changes drastically depending on them. In this paper, the author analyzes the viscous flow (laminar) region. But before that, basics of **Table 3** is explained first in the next section.

3. Basics of Vacuum

3-1 Mean free path

The gas is a group of many molecules. Each of the molecules moves randomly and with various speeds, the average of which is defined by the molecules' type and temperature (at room temperature, ca. 500 ~ 1500 m/s). Since the number of molecules are usually very large (2.7E22 pcs/L, at room temperature), they repeat mutual collisions at a high rate. The average distance between one collision to the next is called the 'mean free path.'

It is known that the mean free path λ [m] is described as follows(3).

$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 P} \fallingdotseq \frac{3.11\times10^{-24}T}{d^2P} \quad \cdots\cdots(1)$$

Here, P [Pa] is the pressure, T [K] is the temperature, d [m] is the diameter of the molecule, and k [J/K] is the Boltzmann's constant.

If one assumes that T = 300 [K] and the gas is nitrogen, then the molecule's diameter d [m] is 0.37 [nm](5),(6).

$$\lambda = 6.8\times10^{-3}/P \quad \cdots\cdots(2)$$

(The value is almost the same, if the gas is atmosphere.)

3-2 Viscous flow and molecular flow

As **Fig. 1 (a)** shows, when the pressure is high, collisions between molecules are dominant. As one can see in **Equation (2)**, when the pressure decreases, the mean free path gets longer, and eventually there is a case when the collisions between molecules are less than those of molecules and the wall (See **Fig. 1 (b)**).

Here, we distinguish those cases as follows:
(a) viscous flow
(b) molecular flow

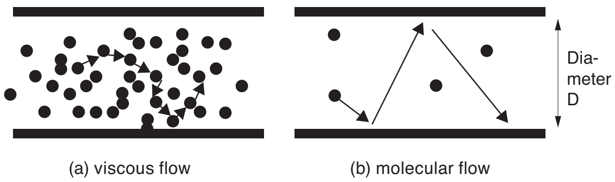

Fig. 1. Viscous vs. molecular flows

3-3 Index of flow: Knudsen number and PD

In **Fig. 1**, when D [m] is defined as the diameter of the pipe (or chamber), one can see that the flow is viscous when λ << D, and is molecular when λ >> D. So if non-dimensional number K = λ/D is introduced, those conditions are K >> 1 and K << 1, respectively. This K is called Knudsen number. In practical use, it is often the case when K < 0.01 is regarded as viscous, and K > 0.3 as molecular.

And when we put these relations into **Equation (2)**, we get

Viscous flow : PD > 0.68 [Pa·m]
Molecular flow : PD < 0.02.

In practical design of the vacuum system, these representations are easier to grasp.

3-4 Conductance

When gas flows in a pipe, resistance caused by it is called 'evacuation resistance,' and its inverse is called 'conductance.' In a schematic diagram as **Fig. 2**, let us call the pressures at both ends P_1 and P_0 [Pa], and flow rate Q [Pam³/s]. Then the conductance C [m³/s] is expressed as

$$Q = C(P_1 - P_0) \quad \cdots\cdots(3)$$

Pipe's conductance both in viscous flow and in molecular flow has been calculated theoretically, as in **Table 4**.

One can see the points:

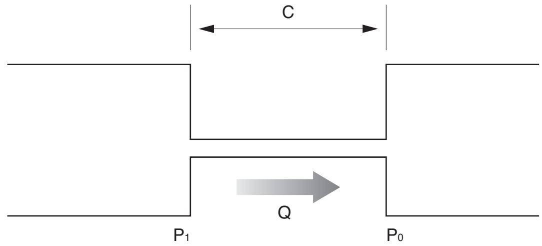

Fig. 2. Schematic diagram of conductance

Table 4. Conductance(2) of (long) pipe

	Viscous flow	Molecular flow
Conductance C [m^3/s]	$\frac{1349 \cdot D^4 \cdot P}{L}$	$\frac{121 \cdot D^3}{L}$

P= $(P_1+P_0)/2$ [Pa], D: Diameter [m], L: Length [m]

· Viscous flow: is proportional to average pressure and 4th power of D.
· Molecular flow: is independent from the pressure, but is proportional to the 3rd power of D.

As for the conductance of the intermediate flow, taken the molecular conductance as 1, an approximate equation(3)

$\frac{1 + 201 \cdot PD + 2647 \cdot (PD)^2}{1 + 236 \cdot PD}$ is advocated. (See **Fig. 3**)

According to **Fig. 3**, it is enough to think that the lower limit of viscous flow is PD ≒ 0.3 ~ 0.4.

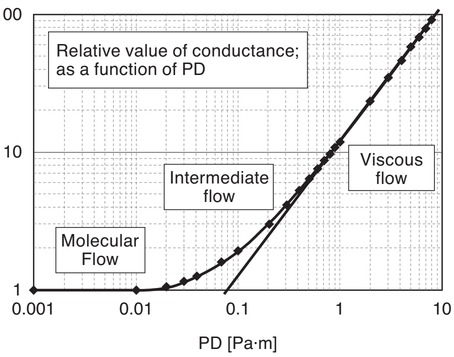

Fig. 3. Relative conductance, as a function of PD

3-5 Evacuation speed & time (simple calculation)

Let us think a simple evacuation system as in **Fig. 4**. We will evacuate the chamber (the volume V [m^3]) by a pump (pumping speed = S [m^3/s]), connected with a pipe (the conductance C [m^3/s]). In elementary texts, the answer is to solve the next **Equation (4)**, by simply omitting the pipe's conductance (since it is large enough), or by thinking that the effective pumping speed Se is about 0.8 S.

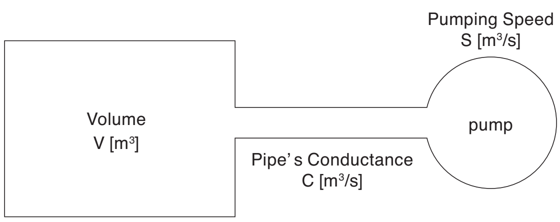

Fig. 4. Simple evacuation system

$$V \cdot \Delta P = -S_e \cdot P \cdot \Delta t \quad \cdots\cdots (4)$$

When it is integrated, we get

$$\ln(P) = -\frac{S_e}{V} \cdot t + const. \quad \cdots\cdots (5)$$

After solving **Equation (5)** with the initial condition t = 0, P = P_0, we will get

$$\begin{cases} P = P_0 \cdot \exp(-S_e \cdot t / V) & \cdots\cdots (6) \\ t = 2.303 \cdot \frac{V}{S_e} \log\left(\frac{P_0}{P}\right) & \cdots\cdots (7) \end{cases}$$

These two **Equations(6),(7)** are simple and clear enough, but they have a problem with the prerequisite.

That is, as described in section 3-4, the pipe's conductance C is proportional to the average pressure. In the above calculation, an assumption is made that the C is large enough, or just its effect is 80%. But what will happen, if it can not be neglected? Or is there any exact answer? In this paper, the author will consider this general and exact case.

4. General Answer to the Evacuation Calculation in Viscous Flow

4-1 Introduction of basic equation

For introducing the basic equation, let us redefine the physical parameters as in **Fig. 5**.

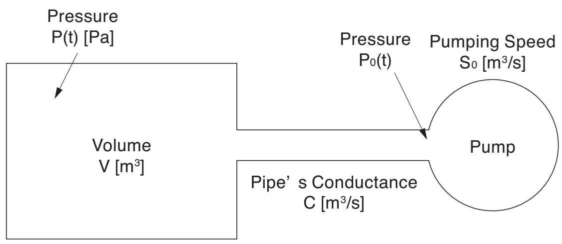

Fig. 5. Evacuation system in viscous flow

As described in section 3-4, in a viscous flow the pipe's conductance C is proportional to the average pressure. So let us call the ratio of the conductance to the pressure 'conductance coefficient' Cc [m^3/sPa].

From **Table 4**, in a room temperature & atmosphere, $C_c = \frac{1349 \cdot D^4}{L}$,where D [m] is the pipe's diameter, L [m] is the length of it.

As in **Fig. 5**, let us define that S_0 is the pumping speed, $P_0(t)$ is the pressure at the pump inlet, $P(t)$ is the pressure of the chamber and is a function of time t. We will set the assumptions below.

- The ultimate pressure of the pump is small enough, and can be neglected.
- The chamber is large enough, so that the pressure in it is uniform, and the volume of the pipe can be neglected.
- Leak and out-gassing in the system can be negligible.
- The flow is viscous, at least in the region of our concern.

Let us try to get $P(t)$, starting $P(t) = P_0$ at $t = 0$, on the conditions above.

When we name the flow rate in the pipe as $Q(t)$ [Pam^3/s], from **Equation (3)** we get,

$$Q(t) = C\cdot(P(t)-P_0(t))$$

$$= \frac{C_c}{2}(P(t)^2-P_0(t)^2)$$

$$\rightarrow P(t)^2-P_0(t)^2 = \frac{2}{C_c}\cdot Q(t) \quad \cdots\cdots\cdots(8)$$

And from the definition of the pumping speed, we get

$$Q(t) = S_0\cdot P_0(t) \quad \cdots\cdots\cdots(9)$$

And, from the material balance that the pressure drop per unit hour in the chamber is equal to the flow rate, we get the next equation.

$$-V\cdot\frac{dP}{dt} = Q(t) \quad \cdots\cdots\cdots(10)$$

When we put **Equation (9)** to **(8)** and **(10)**, we get

$$\begin{cases} P(t)^2-P_0(t)^2 = \dfrac{2}{C_c}S_0P_0(t) & \cdots\cdots\cdots(11) \\ -V\cdot\dfrac{dP}{dt} = S_0P_0(t) & \cdots\cdots\cdots(12) \end{cases}$$

From **Equation (11)**,

$$P_0(t) = -\frac{S_0}{C_c} + \sqrt{\left(\frac{S_0}{C_c}\right)^2 + P(t)^2} \quad \cdots\cdots\cdots(13)$$

And putting **Equation (13)** to **(12)**, we get

$$\frac{-V}{S_0}\cdot\frac{dP}{dt} = \frac{-S_0}{C_c} + \sqrt{\left(\frac{S_0}{C_c}\right)^2 + P(t)^2} \quad \cdots\cdots\cdots(14)$$

Our first aim is to solve this differential **Equation (14)** on the initial condition of $t = 0$, $P(t) = P_0$.

4-2 Deduction of the general solution

Luckily we can solve **Equation (14)** analytically. Here, we put

$$P(t) = \frac{S_0}{C_c}\cdot x(t) \quad \cdots\cdots\cdots(15)$$

We put **Equation (15)** into **(14)** and get

$$\frac{-V}{S_0}\frac{dx}{dt} = -1+\sqrt{1+x(t)^2} \quad \cdots\cdots\cdots(16)$$

Here, we put $\dfrac{V}{S_0} = \tau$ (time constant) $\cdots\cdots\cdots(17)$

$$-\tau\frac{dx}{dt} = -1+\sqrt{1+x(t)^2} \quad \cdots\cdots\cdots(18)$$

After integrating **Equation (18)**, we get

$$\int\frac{dx}{\sqrt{1+x^2}-1} = \int\frac{-dt}{\tau} \quad \cdots\cdots\cdots(19)$$

Left hand side of (19) $= \displaystyle\int\frac{\sqrt{1+x^2}+1}{x^2}dx$

Here, from a formulary[7],

$$= \frac{-\sqrt{1+x^2}}{x} + \ln\left(x+\sqrt{1+x^2}\right) - \frac{1}{x} \quad \cdots\cdots\cdots(20)$$

Right hand side of (19)$=\displaystyle\int\frac{-dt}{\tau} = \frac{-t}{\tau} - A$ (constant) $\cdots(21)$

From **Equations (20)** and **(21)**, we get

$$\frac{-\sqrt{1+x^2}}{x} + \ln\left(x+\sqrt{1+x^2}\right) - \frac{1}{x} = \frac{-t}{\tau} - A \quad \cdots\cdots\cdots(22)$$

Here, $P(t) = \dfrac{S_0}{C_c}x(t)$

Let us solve **Equation (22)**, on the initial condition $t = 0$ and $P(t) = P_0$.

When we put $P_0 = \dfrac{S_0}{C_c}x_0$,we get

$$A = \frac{1+\sqrt{1+x_0^2}}{x_0} - \ln\left(x_0+\sqrt{1+x_0^2}\right) \quad \cdots\cdots\cdots(23)$$

$$\frac{t}{\tau} = \frac{1+\sqrt{1+x^2}}{x} - \ln\left(x+\sqrt{1+x^2}\right) - A \quad \cdots\cdots\cdots(24)$$

$$P(t) = \frac{S_0}{C_c}x(t),\ P_0 = \frac{S_0}{C_c}x_0,\ \tau = \frac{V}{S_0}$$

Note that these are the general solution of the **Equation (22)**.

4-3 Two extreme cases

Since $\tau = \dfrac{V}{S_0}$

has the unit of [s] and appears by the form of t/τ, we can safely assume that it is a 'time constant.' In the same way,

$\dfrac{S_0}{C_c}$

has the unit of [Pa] and so we can assume that it is some kind of typical pressure. From now on, we put

$$\Pi = \frac{S_0}{C_c},$$

and call this as 'transferring pressure.' With this, **Equations (23)** and **(24)** are expressed:

$$\frac{t}{\tau} = \frac{1+\sqrt{1+(P/\Pi)^2}}{(P/\Pi)} - \ln\left(\frac{P}{\Pi} + \sqrt{1+(P/\Pi)^2}\right) - A$$

$$A = \frac{1+\sqrt{1+(P_0/\Pi)^2}}{(P_0/\Pi)} - \ln\left(\frac{P_0}{\Pi} + \sqrt{1+(P_0/\Pi)^2}\right) \quad \cdots\cdots(25)$$

(Note that P is a function of time t.)

We can say that the pressure is normalized by the transferring pressure Π. In general, since P_0 is atmospheric pressure, $P_0 >> \Pi$ can be concluded in many cases.

So $A \approx 1 - \ln\left(\frac{2P_0}{\Pi}\right)$ ············(26) is valid in general.

Here we examine two extreme cases.

【Case 1】 In the case of $P >> \Pi$

(**Equation (26)** holds true automatically.)

$$\frac{t}{\tau} \approx \left\{1 - \ln\left(\frac{2P}{\Pi}\right)\right\} - \left\{1 - \ln\left(\frac{2P_0}{\Pi}\right)\right\} = \ln\left(\frac{P_0}{P}\right) \quad \cdots\cdots(27)$$

Namely, $$P \approx P_0 \exp\left(-\frac{t}{\tau}\right) = P_0 \exp\left(\frac{-S_0 \cdot t}{V}\right) \quad \cdots\cdots(28)$$

【Case 2】 In the case of $P << \Pi$

$$\frac{t}{\tau} \approx \frac{2}{P/\Pi} - \ln(1) - A \approx \frac{2\Pi}{P} \quad \cdots\cdots(29)$$

$$\because \begin{cases} \text{In the case of } P_0 \approx \Pi, A \approx 1+\sqrt{2} - \ln\left(1+\sqrt{2}\right) \cong 1.53 \\ \text{In the case of } P_0 >> \Pi, \dfrac{2\Pi}{P} >> -A \fallingdotseq \ln\left(\dfrac{2P_0}{\Pi}\right) - 1 \end{cases}$$

Namely, $$P \approx \frac{2\Pi \cdot \tau}{t} = \frac{2 \cdot V}{C_c} \cdot \frac{1}{t} \quad \cdots\cdots(30)$$

4-4 Interpretation of the general solution

Let us try to interpret the two cases in the previous section.

1) In the case when the transferring pressure $\Pi = S_0/C_c$ is normal (or moderate). In this case, when one starts evacuation from the atmosphere, first the pressure decreases exponentially to time (as in case 1). When the pressure decreases enough lower than the transferring pressure, the pressure decreases inversely proportional to time, independently to the initial pressure (as in case 2). The pressure when these two cases come across virtually is the transferring pressure Π.
2) In the case when the pipe's diameter is large enough, and so the transferring pressure is small and almost lower than the limit of the viscous flow. In this case, there is no actual case 2 state, but almost all the evacuation time the case 1 happens. That is, the pressure decreases exponentially to time. This is the state that the author described in section 3-5, and also that usual vacuum texts describe in them.
3) On the other hand, there is a case when the pipe's diameter is too small and so the transferring pressure is too high to almost reach the atmosphere. In this case the case 1 does not happen, but all the time of evacuation the pressure decreases inversely proportional to time (case 2). This is the case when the pumping speed is masked by the resistance of the pipe. The evacuation time is the function of only the volume V and the conductance coefficient Cc, and is independent of the initial pressure or pumping speed (Cf. Eq. 30).

In the general evacuation of viscous flow, all three states (1 to 3) can be appeared. We must note that the state 2), as described in texts, is not the only one; in the state 1) and 3), the fact that the region when 'the pressure decreases inversely proportional to time' exists, is very note-worthy.

In **Fig.6** these states are demonstrated graphically. Here, L = 3 [m], V = 10 [L], S_0 = 200 [L/min], P_0 = 1e5 [Pa].

Three typical curves (1 to 3) are plotted on the graph. The lower limit of viscous flow is PD = 0.3 .

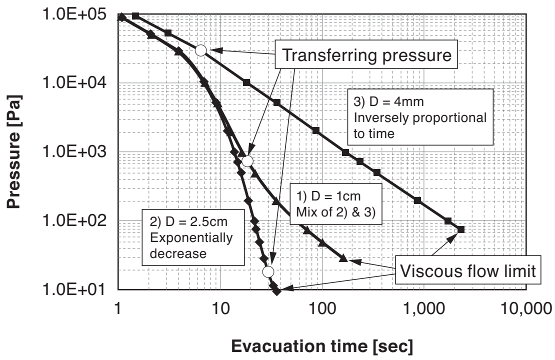

Fig. 6. Calculation of three types of evacuation

4-5 Measured example

Figure 7 shows a measured example.

V = 8.6 [L], S_0 = 180 [L/min], L = 7.5 [m], D = 7.53 [mm]

(3/8" tube, thickness 1 mm), P_0 = 1e5 [Pa] (atmosphere)

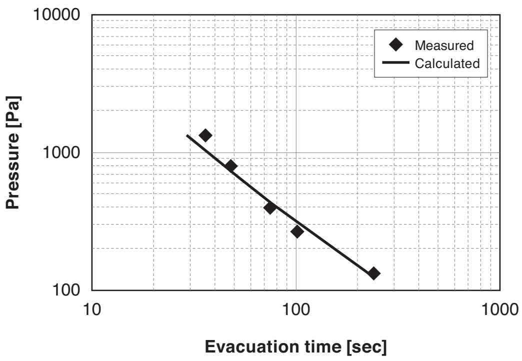

Fig. 7. Measured example

5. Applications and Attentions

5-1 Application

One may think that the case when the pressure decreases inversely proportional to time is meaningless, for the evacuation takes time only. But it is not always so. When one evacuates a chamber with a large pipe from the atmosphere, a turbulent flow occurs at the start. This causes troubles by scattering particles in the chamber in many cases. To prevent this, it is a standard technique to start evacuation slowly with a small pipe (**Fig. 8**).

At this time, one can answer the question "How long does it take to the pressure when the flow comes to laminar, with what size of the pipe is suitable for this?," using equations described above. If one uses too small pipes, it may take too much time. Sometimes there is a case that two channels are necessary for the slow piping.

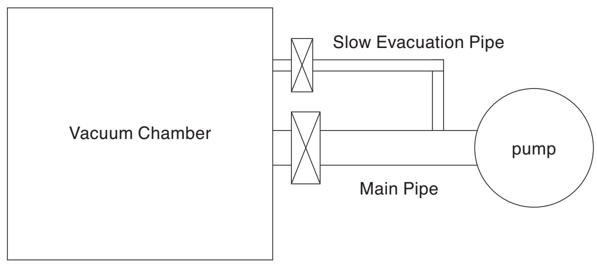

Fig. 8. Example of slow evacuation

5-2 Attention

Figure 9 is a redrawn graph of **Fig. 6**, with the linear horizontal axis and the logarithmic vertical axis.

One may notice that the curve 1) is not linear but asymptotic to the x-axis.

If one believes elementary texts' description that the roughing evacuation curve should be linear in a semi-log graph, it is a serious mistake. One may try to search leakage which actually does not exist. Of course, it is not necessary. In this case, it is simply that the pipe's diameter is too small, if the slow evacuation is not your intention.

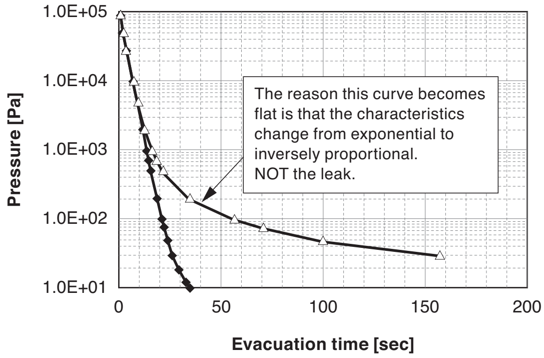

Fig. 9. Evacuation curve with semi-log scale

6. Conclusion

In the world of vacuum industry, descriptions in texts about viscous flow evacuation is limited, probably because it is thought to be rather trivial.

In this paper, the author analyzed it and found that what is thought to be simple is actually not so simple. He introduced the notion of 'transferring pressure' and showed that the evacuation curve in a viscous flow is first exponential, and then inversely proportional to time. In particular, when the pipe's resistance dominates, the fact that "the pressure decreases inversely proportional to time" is simple and important. It is strange that there is virtually no text which clearly points out this fact.

The solution deduced mathematically this time has a rather complicated form, and is not so easy to grasp. But nowadays when computer use is ordinary, it can be a useful tool when used with a spread sheet software for making graphs or simulations.

It is the author's pleasure that this paper be some help for the people concerned with vacuum industries.

References

(1) JIS Z 8126 'Vacuum technology-Vocabulary' p2 (1990)
(2) 'Vacuum handbook (rev. ed.),' pp39-41, Nippon-Shinku-Gijutsu Co., Ltd. (1982)
(3) Yoshitaka Hayashi, 'Introduction to Vacuum Technology,' pp18-19, p13, p21, Nikkan-Kogyo Shinbun Co., Ltd. (1987)
(4) URL http://www.nucleng.kyoto-u.ac.jp/People/ikuji/edu/vac/app-A/conduct.html
(5) URL http://www.nucleng.kyoto-u.ac.jp/People/ikuji/edu/ebeam/lennard.html
(6) URL http://www.nucleng.kyoto-u.ac.jp/People/ikuji/edu/vac/app-A/mfp.html
(7) Yoshihiko Ootsuki (Translator), 'Mathematical Formulary,' pp86-88, Maruzen (1983)

(8) John F. Ohanlon, 'Vacuum Technology Manual,' Sangyo Tosho Co., Ltd. (1985)
(9) Katsuya Nakagawa, 'Vacuum Technologies, engineers' manual,' Ohm-sha Co., Ltd. (1985)
(10) Hiroshi Nakagawa, 'What to do against vacuum troubles; learning from failures,' Trend Books (1982)
(11) Kazuo Tanida, 'Vacuum system engineering –Basics and applications,' Yokendo (1977)

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